Example - Confidence interval

Based on the sample mean and variance estimates for the eight–analysts example, generate a 95% confidence interval for the population mean (μ). Recall from the example:

$$\bar{x} = 12.6 \qquad s = 2.12 \qquad n = 8$$

We know that the variable

$$t_{n-1} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

is distributed as a t with ν degrees of freedom. With 95% confidence:

$$P(-t_{\nu,\frac{\alpha}{2}} \leq t_\nu \leq t_{\nu,\frac{\alpha}{2}}) = 1 - \alpha$$

which, for $\nu = n - 1 = 7$, and $\alpha = 0.05$, becomes:

$$P(-t_{7,0.025} \leq t_\nu \leq t_{7,0.025}) = 0.95$$

From the definition of out t–variable:

$$P(-t_{7,0.025} \leq \frac{\bar{x} - \mu}{s/\sqrt{n}} \leq t_{7,0.025}) = 0.95$$

which, after moving to both sides $\bar{x}$, and $s/\sqrt{n}$, we get:

$$P\left[-t_{7,0.025}(s/\sqrt{n}) - \bar{x} \leq -\mu \leq t_{7,0.025}(s/\sqrt{n}) - \bar{x}\right] = 0.95$$

and after multiplying by -1 the expression becomes:

$$P\left[t_{7,0.025}(s/\sqrt{n}) + \bar{x} \leq \mu \leq -t_{7,0.025}(s/\sqrt{n}) + \bar{x}\right] = 0.95$$

or,

$$\boxed{P\left[\bar{x} - t_{7,0.025}(s/\sqrt{n}) \leq \mu \leq \bar{x} + t_{7,0.025}(s/\sqrt{n})\right] = 0.95}$$

and replacing the values of $\bar{x}$, s, n, and $t_{7,0.025} = 2.365$, we get the following 95% confidence interval:

$$\boxed{10.827 < \mu < 14.372}$$