PROBLEM:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{4} (k+1)x[n-k]$$

The input to this system is *unit step* signal, denoted by $u[n]$, i.e., $x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Determine the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Use convolution to compute $y[n]$, over the range $-5 \leq n \leq \infty$, when the input is $u[n]$. Make a plot of $y[n]$ vs. n. (Hint: you might find it useful to check your results with MATLAB's `conv( )` function.)

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

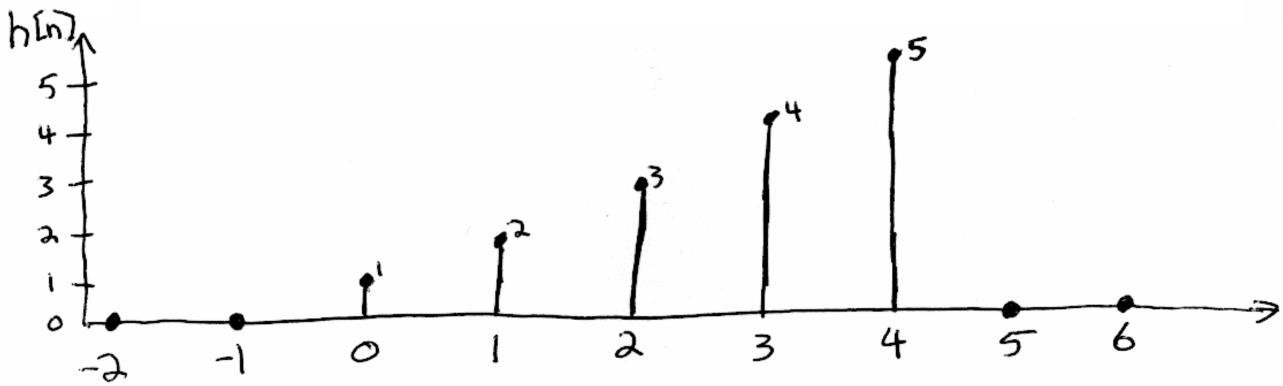
SOLUTION

a) $y[n] = 1x[n] + 2x[n-1] + 3x[n-2] + 4x[n-3] + 5x[n-4]$

Filter coefficients $\boxed{b_0 = 1 \quad b_1 = 2 \quad b_2 = 3 \quad b_3 = 4 \quad b_4 = 5}$

($b_n = 0$ for $n < 0$ and $n > 4$)

b) $h[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 5\delta[n-4]$



c) $y[n] = \sum_{k=0}^{4} h[k]\, u(n-k)$

n	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
u(n)	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
h(n)	0	0	0	0	0	1	2	3	4	5	0	0	0	0	0	0
h(0) u(n)	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
h(1) u(n-1)	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2
h(2) u(n-2)	0	0	0	0	0	0	0	3	3	3	3	3	3	3	3	3
h(3) u(n-3)	0	0	0	0	0	0	0	0	4	4	4	4	4	4	4	4
h(4) u(n-4)	0	0	0	0	0	0	0	0	0	5	5	5	5	5	5	5
y[n]	0	0	0	0	0	1	3	6	10	15	15	15	15	15	15	15

y[n] for $n < 0$

y[0] y[1] y[2] y[3]

y[n] for $n \geq 4$