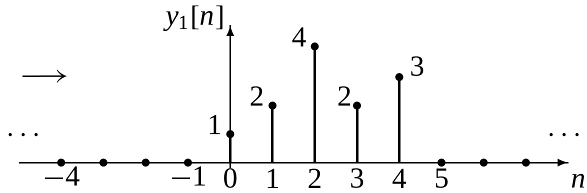
PROBLEM:

Answer the following questions about the time-domain response of FIR digital filters:

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

(a) When tested with an input signal that is the sum of two shifted impulses $x_1[n] = \delta[n+1] + \delta[n-1]$, the observed output from the filter is the signal $y_1[n]$ shown below:

$x_1[n] = \delta[n+1] + \delta[n-1] \quad \longrightarrow$



Determine the filter coefficients $\{b_k\}$ of the difference equation for the FIR filter.

(b) If the input signal is

$$x[n] = \begin{cases} 0 & \text{for } n < 0 \\ (-1)^n & \text{for } n = 0, 1, 2, 3 \\ 0 & \text{for } n > 3 \end{cases}$$

use linearity and time-invariance to determine the output signal $y[n]$ for all n. Give your answer as either a plot or a table of values.

(c) Finally, determine the impulse response of the system. This might be difficult, because you are essentially being asked to solve the following convolution equation:

$$x_1[n] * h[n] = y_1[n]$$

for $h[n]$. In general, it is not always possible to solve such an equation.
Note: The convolution equation can be regarded as a set of simultaneous linear equations in the unknown impulse response values $h[n]$.

(d) Is the filter *causal*? Use the appropriate property of the impulse response that guarantees causality.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

a) Note that $x_1[n]=0$ for all n except $n=-1$ and $n=1$. Thus, given $y_1[n]=\sum_{k=0}^{M} b_k x_1[n-k]$, we may write

$$
\begin{aligned}
y_1[-1] &= & & b_0 x_1[-1] \\
y_1[0] &= & & b_1 x_1[-1] \\
y_1[1] &= b_0 x_1[1] &+& b_2 x_1[-1] \\
y_1[2] &= b_1 x_1[1] &+& b_3 x_1[-1] \\
y_1[3] &= b_2 x_1[1] &+& b_4 x_1[-1] \\
y_1[4] &= b_3 x_1[1] &+& b_5 x_1[-1] \\
&\vdots & & \vdots
\end{aligned}
$$

Substituting values for y_1 and x_1 gives

$$
\left\{
\begin{aligned}
0 &= b_0 & &\longrightarrow & b_0 &= \boxed{0} \\
1 &= b_1 & &\longrightarrow & b_1 &= \boxed{1} \\
2 &= b_0 + b_2 & &\longrightarrow & b_2 &= 2 - b_0 = 2 - 0 = \boxed{2} \\
4 &= b_1 + b_3 & &\longrightarrow & b_3 &= 4 - b_1 = 4 - 1 = \boxed{3} \\
2 &= b_2 + b_4 & &\longrightarrow & b_4 &= 2 - b_2 = 2 - 2 = 0 \\
3 &= b_3 + b_5 & &\longrightarrow & b_5 &= 3 - b_3 = 3 - 3 = 0 \\
0 &= b_4 + b_6 & &\longrightarrow & b_6 &= -b_4 = 0 \\
0 &= b_5 + b_7 & &\longrightarrow & b_7 &= -b_5 = 0 \\
&\vdots & & & &\vdots
\end{aligned}
\right.
$$

all other $b_k = 0$

$$\boxed{M=3,\ \{b_0, b_1, b_2, b_3\} = \{0, 1, 2, 3\}}$$

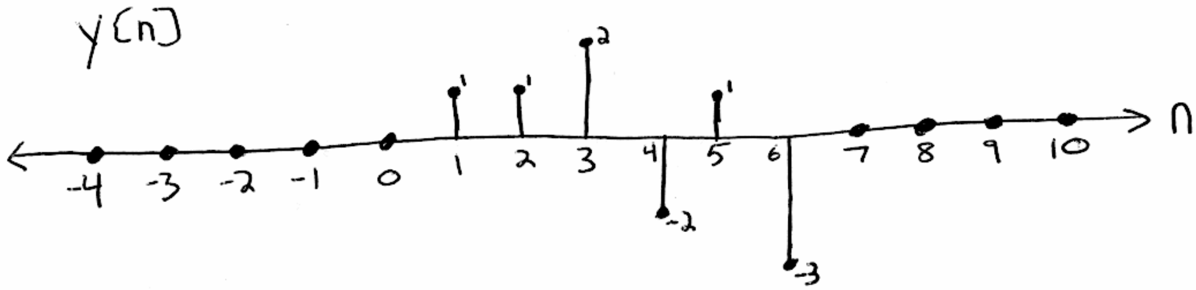
b) $x[n] = (\delta[n] + \delta[n-2]) - (\delta[n-1] + \delta[n-3])$

$= x_1[n-1] - x_1[n-2]$

By LTI properties we therefore have

$y[n] = y_1[n-1] - y_1[n-2]$

n	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
$y_1[n-1]$	0	0	0	0	0	1	2	4	2	3	0	0	0
$-y_1[n-2]$	0	0	0	0	0	0	-1	-2	-4	-2	-3	0	0
$y[n]$	0	0	0	0	0	1	1	2	-2	1	-3	0	0



c) $h[n] = b_0\delta[n] + b_1\delta[n-1] + b_2\delta[n-2] + b_3\delta[n-3]$

$= \boxed{\delta[n-1] + 2\delta[n-2] + 3\delta[n-3]}$

d) Filter is $\boxed{\text{causal}}$ because $h[n] = 0$ for all $n < 0$.