PROBLEM:

An FIR filter is described by the difference equation:

$$y[n] = 3x[n] + 2x[n-3] - 3x[n-5]$$

(a) Find its impulse response $h[n]$ and plot versus n.

(b) Let $x[n]$ be the complex exponential

$$x[n] = 3e^{j(0.4\pi n - \pi/2)} \qquad \text{for all } n$$

Then it is possible to express the output $y[n]$ in the form

$$y[n] = Ae^{j(\omega_0 n + \phi)}$$

Determine the numerical values of A, ϕ and ω_0.

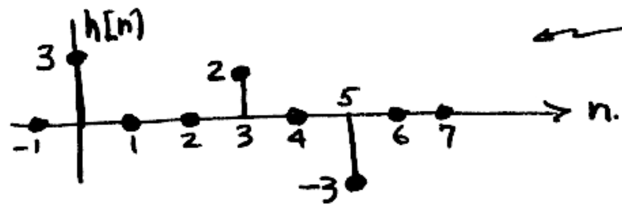
McClellan, Schafer and Yoder, *Signal Processing First,* ISBN 0-13-065562-7.

SOLUTION

(a) $y[n] = \underset{b_0=3}{3}x[n] + \underset{b_3=2}{2}x[n-3] - \underset{b_5=-3}{3}x[n-5]$

Let $x[n] = \delta[n]$

$h[n] = 3\delta[n] + 2\delta[n-3] - 3\delta[n-5]$



h[n] will "read out" the filter coeffs.

(b) $x[n] = 3e^{-j\pi/2}e^{j0.4\pi n}$

$$y[n] = \left(\mathcal{H}(\hat{\omega})\Big|_{\hat{\omega}=0.4\pi}\right)x[n]$$

If we use the frequency response we eval $\mathcal{H}(\hat{\omega})$ at 0.4π

$$\mathcal{H}(\hat{\omega}) = 3 + 2e^{-j3\hat{\omega}} - 3e^{-j5\hat{\omega}}$$

$$= 3 + 2e^{-j1.2\pi} - 3e^{-j2\pi} = 3 + 2e^{-j1.2\pi} - 3 = 2e^{-j1}$$

$$\Rightarrow y[n] = (2e^{-j1.2\pi})3e^{-j\pi/2}e^{j0.4\pi n} = 6e^{-j1.7\pi}e^{j0.4\pi n}$$

$$\boxed{A=6,\ \varphi=-1.7\pi,\ \hat{\omega}_0 = 0.4\pi}$$

We get the same result if we just plug into the difference equation:

$$y[n] = 9e^{-j\pi/2}e^{j0.4\pi n} + 6e^{-j\pi/2}e^{j0.4\pi(n-3)} - 9e^{-j\pi/2}e^{j0.4\pi(n-5)}$$

$$= 3e^{-j\pi/2}e^{j0.4\pi n}\underbrace{\left(3 + 2e^{-j1.2\pi} - 3e^{-j2\pi}\right)}_{\mathcal{H}(\hat{\omega})\text{ at }\hat{\omega}=\hat{\omega}_0=0.4\pi}$$