PROBLEM:

For a particular linear time-invariant system, when the input is

$$x_1[n] = 4u[n] = \begin{cases} 0 & n < 0 \\ 4 & n \geq 0 \end{cases}$$

the corresponding output is

$$y_1[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4u[n-3] = \begin{cases} 0 & n < 0 \\ 1 & n = 0 \\ 2 & n = 1 \\ 3 & n = 2 \\ 4 & n \geq 3 \end{cases}$$

(a) Using the concepts of linearity and time-invariance, determine the impulse response of the system.

(b) The system is an FIR filter—determine the filter coefficients and the length of the filter.

(c) State a general procedure for deriving the impulse response of a LTI system from a measurement of its step response, i.e., if $s[n]$ is the step response of a LTI system, what simple operations can be done to $s[n]$ to produce the impulse response $h[n]$.

(d) Using the concepts of linearity and time-invariance, determine the output signal when the input signal is $x_2[n] = 7u[n-1] - 7u[n-4]$. Give your answer as a formula expressing $y_2[n]$ in terms of known sequences or as an equation for each value of $y_2[n]$ for $-\infty < n < \infty$.

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.

SOLUTION

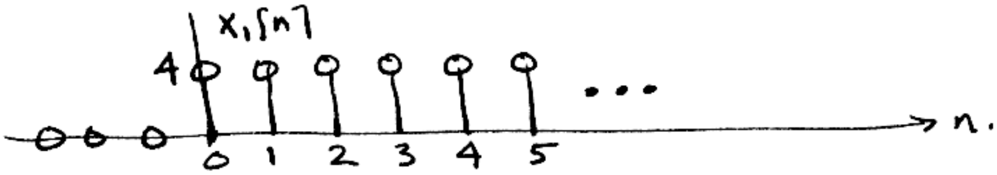
(a) $x_1[n] \longrightarrow y_1[n]$ is a known input-output pair

We need to express $\delta[n]$ in terms of $x_1[n]$ which means we are allowed to shift and scale $x_1[n]$ to create $\delta[n]$. In other words, can we find α_1, n_1, α_2 and n_2 so that

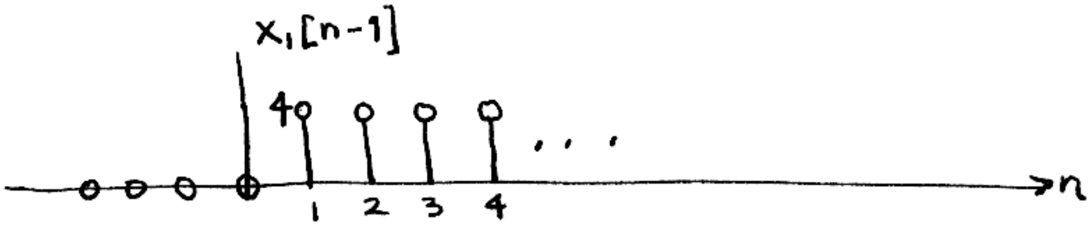
$$\delta[n] \stackrel{?}{=} \alpha_1 x_1[n-n_1] + \alpha_2 x_1[n-n_2]$$

Maybe it is easiest to see from plots.

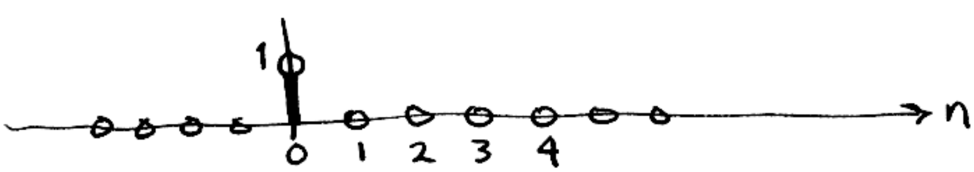
Make a plot of $x_1[n]$



Make a plot $x_1[n]$ shifted by one (delay).



If we want to combine these two plots to get $\delta[n]$



Then, clearly we need to subtract, and divide by 4.

<u>Thus</u> $\quad \delta[n] = \frac{1}{4} x_1[n] - \frac{1}{4} x_1[n-1]$

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Prob (cont)

(a, cont) Now we can use linearity

If $\delta[n] = \frac{1}{4} x_1[n] - \frac{1}{4} x_1[n-1]$

then $h[n] = \frac{1}{4} y_1[n] - \frac{1}{4} y_1[n-1]$

BECAUSE
$x_1[n] \longrightarrow y_1[n]$
$x_1[n-1] \longrightarrow y_1[n-1]$

MAKE A TABLE TO COMPUTE $h[n]$.

n	$n<0$	0	1	2	3	4	$n \geq 5$
$y_1[n]$	0	1	2	3	4	4	4 .....
$y_1[n-1]$	0	0	1	2	3	4	4 ...
$h[n]$	0	1/4	1/4	1/4	1/4	0	0 ....

$h[n] = \frac{1}{4}(3) - \frac{1}{4}(2) = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

(b) For an FIR filter the impulse response will "read out" the coefficients.

$\Rightarrow \{b_k\} = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\}$ $\quad L = 4$ is FILTER LENGTH

(c) Since $\delta[n] = u[n] - u[n-1]$, if we have the step response $s[n]$, the impulse response $h[n]$ can be constructed via: $h[n] = s[n] - s[n-1]$.

(d) Write $x_2[n]$ in terms of $x_1[n]$

$$x_2[n] = \frac{7}{4} x_1[n-1] - \frac{7}{4} x_1[n-4].$$

$$\Rightarrow y_2[n] = 7/4\, y_1[n-1] - 7/4\, y_1[n-4]$$ USING L.T.I.

MAKE A TABLE

n	$n<0$	0	1	2	3	4	5	6	7	8	$n \geq 9$
$y_1[n-1]$	0	0	1	2	3	4	4	4	4	4	4 ...
$y_1[n-4]$	0	0	0	0	0	1	2	3	4	4	4 ...
$y_2[n]$	0	0	$\frac{7}{4}$	$\frac{7}{2}$	$\frac{21}{4}$	$\frac{21}{4}$	$\frac{7}{2}$	$\frac{7}{4}$	0	0	0 ...