



PROBLEM:

In each of the following cases, use known Fourier transform pairs together with Fourier transform properties to complete the following Fourier transform pair relationships:

$$(a) \quad x(t) = u(t + 3)u(3 - t) \quad \iff \quad X(j\omega) =$$

$$(b) \quad x(t) = \sin(4\pi t) \sin(50\pi t) \quad \iff \quad X(j\omega) =$$

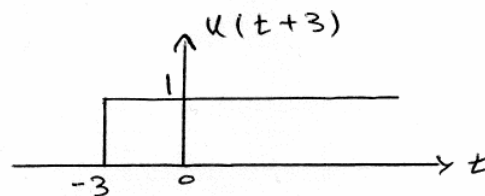
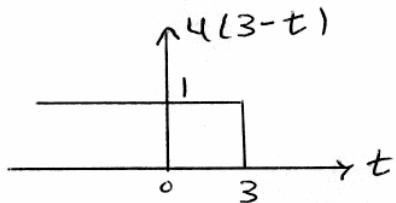
$$(c) \quad x(t) = \frac{\sin 4\pi t}{\pi t} \sin(50\pi t) \quad \iff \quad X(j\omega) =$$

$$(d) \quad x(t) = \quad \iff \quad X(j\omega) = \frac{\sin^2(200\omega)}{\omega^2}$$

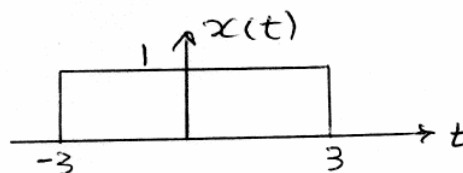
$$(e) \quad x(t) = \quad \iff \quad X(j\omega) = \cos^2(\omega)$$



(a)



$$x(t) = u(t+3) \cdot 4(3-t) \rightarrow$$



$$T_{0/2} = 3 \rightarrow X(j\omega) = \frac{\sin(3\omega)}{\omega/2}$$

(b)

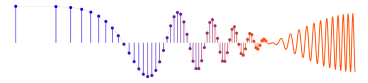
t-domain ω-domain

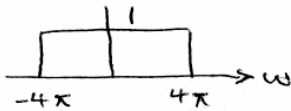
note $\sin 4\pi t \longleftrightarrow \frac{\pi}{j} \delta(\omega - 4\pi) - \frac{\pi}{j} \delta(\omega + 4\pi)$

Convolution property of the Fourier Transform:

$$X(j\omega) = \frac{1}{2\pi} \left\{ \frac{\pi}{j} \delta(\omega - 4\pi) - \frac{\pi}{j} \delta(\omega + 4\pi) \right\} * \left\{ \frac{\pi}{j} \delta(\omega - 50\pi) - \frac{\pi}{j} \delta(\omega + 50\pi) \right\}$$

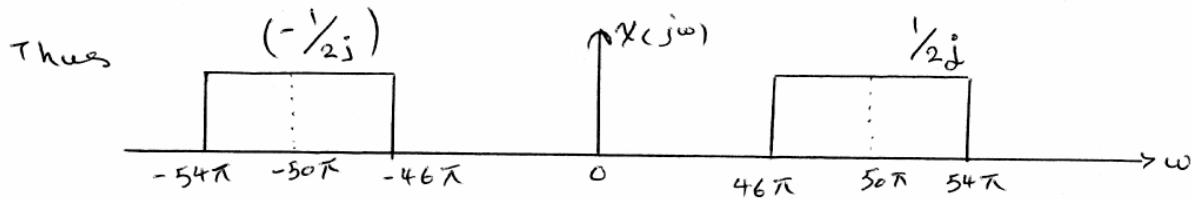
$$X(j\omega) = \frac{\pi}{2} \delta(\omega - 46\pi) + \frac{\pi}{2} \delta(\omega + 46\pi) - \frac{\pi}{2} \delta(\omega - 54\pi) - \frac{\pi}{2} \delta(\omega + 54\pi)$$



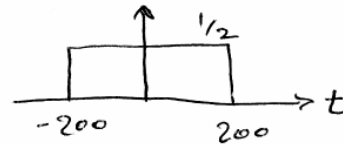
(c) Note $\frac{\sin 4\pi t}{\pi t} \longleftrightarrow$ 

$$X(j\omega) = \frac{1}{2\pi} \left\{ \begin{array}{c} \text{rect}(\omega) \\ \text{from } -4\pi \text{ to } 4\pi \end{array} \right\} * \left\{ \frac{\pi}{j} \delta(\omega - 50\pi) - \frac{\pi}{j} \delta(\omega + 50\pi) \right\}$$

convolution

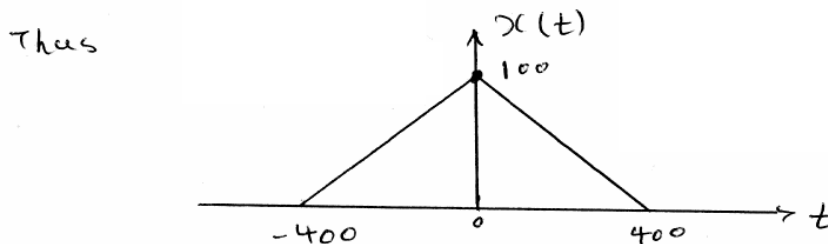


$$X(j\omega) = \begin{cases} \frac{1}{2j} & 46\pi \leq \omega \leq 54\pi \\ -\frac{1}{2j} & -54\pi \leq \omega \leq -46\pi \\ 0 & \text{else} \end{cases}$$

(d) Note: $\frac{1}{2} \frac{\sin(200\omega)}{\omega/2} \longleftrightarrow$ 

$$\frac{\sin^2(200\omega)}{\omega^2} \longleftrightarrow \left\{ \begin{array}{c} \text{rect}(t) \\ \text{from } -200 \text{ to } 200 \end{array} \right\} * \left\{ \begin{array}{c} \text{rect}(t) \\ \text{from } -200 \text{ to } 200 \end{array} \right\}$$

convolution





$$(e) \cos \omega \longleftrightarrow \frac{1}{2} \{ \delta(t-1) + \delta(t+1) \}$$

$$\cos^2 \omega \longleftrightarrow \frac{1}{2} \{ \delta(t-1) + \delta(t+1) \} * \frac{1}{2} \{ \delta(t-1) + \delta(t+1) \}$$

$$x(t) = \frac{1}{4} \{ \delta(t-2) + \delta(t+2) + 2\delta(t) \}$$