PROBLEM:

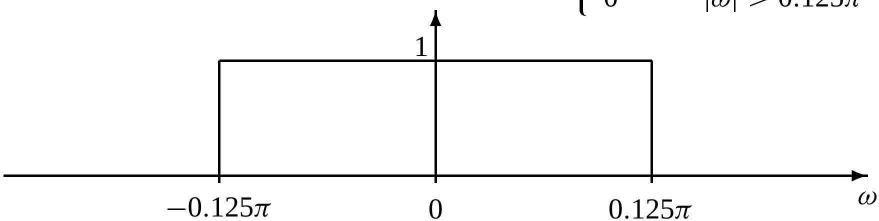
The input to the LTI system shown below is a periodic signal $x(t)$ that has a period $\boxed{T_0 = 20}$ seconds. The Fourier series representation for the input $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \text{where} \qquad a_k = \begin{cases} -1 & k = 0 \\ \dfrac{\sin(\pi k/2)}{2\pi k} & k \neq 0. \end{cases}$$

(a) What is the fundamental frequency ω_0 of the input signal $x(t)$? $\boxed{\omega_0 = \underline{\qquad\qquad} \text{ rad/sec}}$

(b) Suppose that the frequency response of the system is an ideal lowpass filter as illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| < 0.125\pi \\ 0 & |\omega| > 0.125\pi \end{cases}$$



Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. (Your answer should be expressed in terms of only real quantities). **Justify your answer** by sketching the spectrum of $y(t)$ (or its Fourier transform).

(c) Draw the spectrum of the output signal superimposed on the plot of $H(j\omega)$.

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SOLUTION

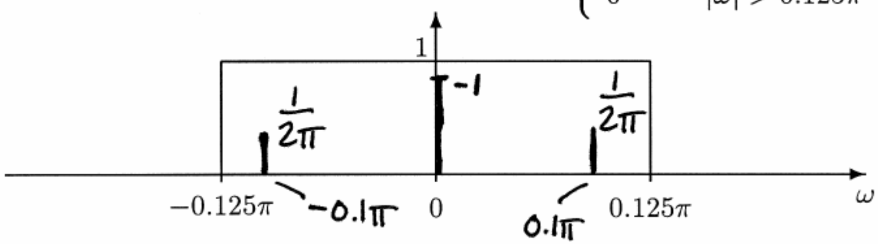
The input to the LTI system shown below is a periodic signal $x(t)$ that has a period $\boxed{T_0 = 20}$ seconds. The Fourier series representation for the input $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \text{where} \qquad a_k = \begin{cases} -1 & k = 0 \\ \dfrac{\sin(\pi k/2)}{2\pi k} & k \neq 0. \end{cases}$$

(a) What is the fundamental frequency ω_0 of the input signal $x(t)$? $\boxed{\omega_0 = \frac{2\pi}{20} = \frac{\pi}{10} \text{ rad/sec}}$

(b) Suppose that the frequency response of the system is an ideal lowpass filter as illustrated below.

$$H(j\omega) = \begin{cases} 1 & |\omega| < 0.125\pi \\ 0 & |\omega| > 0.125\pi \end{cases}$$



Give an equation for the output of the system, $y(t)$, that is valid for $-\infty < t < \infty$. (Your answer should be expressed in terms of only real quantities). **Justify your answer** by sketching the spectrum of $y(t)$ (or its Fourier transform).

$$y(t) = -1 + \frac{1}{2\pi} e^{j\frac{\pi}{10}t} + \frac{1}{2\pi} e^{-j\frac{\pi}{10}t}$$

$$\boxed{y(t) = -1 + \frac{1}{\pi}\cos\left(\frac{\pi}{10}t\right)}$$

(c) Draw the spectrum of the output signal superimposed on the plot of $H(j\omega)$.

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